Some comments on 10,000 year return period rainfall

1 Introduction

The primary objective of this report is to address specific questions posed by the DEFRA and Reservoir Safety Working Group via KBR namely:

- Are the results generated by the FEH software (IH, 1999) to derive 10,000 yr return period rainfall, by extrapolation of relationships held to hold up to 2,000 yr return periods, valid? Is the methodology technically defensible?
- If not, can the method be improved to cover this range of return periods?
- If it can be improved, what is needed in the way of revised analysis?
- If it cannot be improved what options are there for derivation of such extreme rainfalls?
- If the method is valid and meaningful, how do we account for the contradiction with PMP?

In the report, some general points are first made, and then the key issues connected with the FEH method are discussed. A final section gives summary answers to the questions. An Appendix discusses some more detailed statistical points; these are to be regarded as preliminary suggestions to whoever may take the work forward, rather than as part of the Report as such.

2 Some general issues

2.1 Nature of extrapolation

Any prediction of extremely rare events is unavoidably fragile. Three criteria can be applied to study any proposed method. One is internal consistency. This ensures that different cases, in the present case different catchments, are treated in a broadly similar way even if nominal target return periods are systematically over- or under-achieved. The second is that all reasonably reliable sources of information are exploited. The third, and particularly important, is that the estimates should, as far as feasible, be checked against empirical experience, limited though that may be.

2.2 Comparison of different procedures

Given the uncertainties involved in any extrapolation it is to be expected that different procedures will give somewhat different answers. Where there are two different reasonably good procedures there are two broad approaches to using both, when both are available. One is to attempt a synthesis and the other is to choose the "better" approach and then to use the other as a check, investigating in more detail cases where there is a major discrepancy.

2.3 Overall view

Extreme rainfalls are only one component in the overall assessment of dam safety. All components, including for example the rainfall-runoff model, contribute uncertainty. It is desirable to look at the procedure as a whole, to form some estimate, even if only very approximate, of the relative importance of uncertainties in the various steps of the procedure, so as to concentrate study on the critical points of the whole procedure.

2.4 Terminology

A recent ICE report (ICE, 2001) recommended abandoning the term *Return period* which indeed is misleading. While I appreciate the attractions of sticking with long-established and familiar terminology, it is suggested here working primarily with a risk per 10^3 dam yr. Thus the return period of 10^{-4} yr would translate into a rate of 0.1 per 10^3 dam yr, so that with roughly 2×10^3 sites under consideration this would correspond to 10 expected exceedances in 50 yr in the relevant catchments. Note that this interpretation does not depend directly on temporal or spatial dependence, although the distribution in time and space would be so dependent; in one extreme scenario one might wait 5×10^3 yr before seeing 10^3 exceedances simultaneously! This is discussed further in Section 2.5. The choice of 10^3 dam yr as a unit of exposure has the merit of relating risk to a tangible number of occurrences in the U.K. context. The object of such a change is mainly to relate the ideas as firmly as possible to potentially observable events, rather than to apparently long time periods. The issue is, of course, only terminological.

There is the important further point that the values given by FEH notionally give at a *particular storm duration*, the rainfall exceeded with specified probability per year. Data about individual extreme storms typically give rainfalls at a specified duration, chosen in the light of the profile of the particular storm to give an extreme occurrence. The probability per 10^3 yr that a storm will occur exceeding its specified level at some duration or other is greater than, and maybe appreciably greater than, the nominated probability. This affects primarily the interpretation of extreme events rather than calculations about reservoir safety, which will depend on behaviour at durations having most impact in the light of the rainfall-runoff model used.

2.5 Assessment of uncertainty

It might be argued that these probabilities (or return periods) should have measures of uncertainty attached to them. The estimates given by any method are subject to effects arising from the relatively small amount of data, both determined by the number of gauges and the shortness of some of the records. They are also subject to systematic errors arising from the particular method of extrapolation used. The first can be assessed quantitatively, at least approximately; the second can only be addressed by sensitivity analysis and by the comparison of alternative procedures. On general grounds it seems likely that the systematic errors may be the more important even though the random errors are certain to be very appreciable. It is important to clarify where the critical sources of uncertainty lie. Nevertheless there are two key issues:

- what direct practical use could be made of the uncertainty of predictions concerning what is already in effect a probability?
- to the extent that a single probability referring to a single site at a single duration is involved, only a single probability need be given; an

average of probabilities is a probability!

For these reasons, while it is very desirable to analyse sources of uncertainty in a few typical cases, it does not seem feasible to recommend additional measures of uncertainty for routine use. Note, however, that if it were decided to use some combination of FEH and PMP then assessment of their relative precision would be needed as the basis for the synthesized method.

2.6 Climate change

Some consideration is needed of the effect of climate change, bearing in mind its differential pattern across the UK and of the effect of trends in mean level and in variability; in some circumstances the latter could be the more critical. For a recent review of the evidence for the UK, see Osborn and Hume (2002).

2.7 FSR

I have not in this report considered a comparison of FEH with FSR. The former is reported to give the more realistic representation of geographical variation, is more elaborate methodologically (not necessarily a good thing, of course) and particularly importantly is based on more extensive and more recent data.

2.8 Some recommendations

Recommendation 2.1. The relative contribution of various kinds of uncertainty to the final estimates directly addressing Reservoir Safety should be assessed in some typical cases. These uncertainties include those concerned *with the rainfall-run off model, and its indirect treatment of nonlinearities, as well as those concerning rainfall.*

Recommendation 2.2. The potential impact of climate change on estimates should be considered, in the first place by simple sensitivity analyses imposing various trends in mean and dispersion of daily rainfall and examining the effect on current estimates, both by PMP and by FEH methods. This might lead on to assessment in the light of models of climate change. This connects strongly with the work of the Babtie Group (2002).

3 The available information

3.1 Discussion

FEH used exclusively data from rain gauges with at least 9 yr of records. There are the following additional sources of information:

- Values of PMP when available
- Reports of extreme rainfall events
- Reports of dam spills

The first is discussed in more detail in Section 4.

Volume 2 of FEH lists 10 major rainfall events; a list referring to an earlier period is in FSR. For each event the estimated return period (probability of exceedance) found by the FEH method (one presumes) at a retrospectively selected duration is given. It is hard to interpret these return periods, in part because recording and selection of these events weights them towards more populated and accessible areas and some arbitrariness may well be involved in their selection; moreover the choice of relevant duration in effect distorts the probabilities . Nevertheless

- the extremes are widely separated in time and space
- the probabilities quoted seem rather high, i.e. the return periods perhaps rather low
- the events refer mostly to relatively populated areas and may in that way be atypical.

On the second point, note that the number of areas "at risk" for inclusion is ill-defined. A very rough and arbitrary estimate obtained by dividing the total area of England, Scotland and Wales by 25 km^2 is roughly 10^3. In the 20 yr period covered one would therefore expect one or two events in the probability range in question; in fact the highest return period quoted is 1.2×10^3 yr, but this is regarded as being based on suspect data (and refers to an unusually long storm duration of 9 days). The next largest return period is 0.6×10^3 yr. This all suggests that the FEH values are very roughly in line with experience but that if anything there is some implication that extreme rainfall levels are being overestimated. Collier et al (2002, p.27) compared extreme levels of a number of storms with various predictions and implied a similar conclusion, but it is certainly not firmly established.

The form of spatial dependence at extreme levels is important for the FEH (and other methods) and there may be valuable information in the patterns of rainfall on the days that individual sites experience especially critical events. In only one of the events described does it seem that very extreme events were widespread.

Spills may be less directly relevant to the estimation of extreme rainfalls but the occurrence of minor spills when notionally the probability of any kind of spill is small is perhaps warning of sensitivity to more extreme rainfall events at that specific site. Some consideration should be given to the level of spill that might give warning of more major events and which therefore would be worth monitoring.

3.2 Some recommendations

Recommendation 3.1. Further analysis of major rainfall events should be made, extending the study of Collier et al (2002), in particular to examine the spatial pattern of rain at times at or near to those events and the storm type and also to examine possible biases arising from the mode of selection of the storms analysed.

Recommendation 3.2. The recording and analysis of information about minor spills should be considered.

Recommendation 3.3. In some typical cases the uncertainties involved in estimating small probabilities (large return periods) via FEH and via PMP should be assessed, with particular reference to the relative importance of largely random errors of estimation and systematic errors. One role of such uncertainty measures is discussed in the Appendix.

4 Role of PMP

PMP is not used in the FEH assessment. The name is potentially misleading in suggesting that a physical maximum is being given, even though the qualifier *Probable* warns against that. The WMO (1986) manual on the calculation of PMP gives a somewhat perfunctory warning about the inevitable and appreciable uncertainties involved. It suggests that in principle judgemental error bounds could be calculated based on an assessment of the various steps in the estimation but states, and maybe implicitly approves, the absence of such limits in applications. Green et al (2002) have, however, attacked this issue in an Australian context and reviewed earlier work on the uncertainty in PMP, via estimation of the annual probability that it will be exceeded, noting dependence on storm type. The essence is to attach probabilities, in particular to the storm transposition process used in developing PMP. They outline a new method which they hope will reduce the uncertainty in the estimated probability of exceeding PMP; in an earlier method the probabilities had ranges of uncertainty of two orders of magnitude. Collier et al (2002) estimated that PMP corresponds for some storms in the UK to probability levels of about 10^4 yr in line with the approximate agreement of PMP and FEH values. The literature on this is confusing, both in the wide range of return periods quoted, from 10^4 or less to 10^9 and on the method of calculation involved. Some more technical comments on this issue are in an Appendix.

If we take as accepted the broad physical soundness of the considerations entering PMP, a key issue concerns is whether such information used that is not already implicitly involved in the FEH method adds substantial precision. This is not the same question as whether PMP is better or worse than FEH.

The discrepancies between PMP and FEH extrapolated values seem in most cases relatively minor considering the massive assumptions involved in such calculations. An initial study (Macdonald and Scott, 2000) showed some discrepancies of a factor of 1.3 or more, with FEH in most cases exceeding PMP; a rather more extensive study with more sites (DETR, 2000) gave a geometric mean discrepancy of 1.1 with appreciable variation around that. The cases where these discrepancies are large may throw some light on the criticality of assumptions underlying the various methods. Discrepancies are likely to depend, perhaps quite strongly, on the probability level involved. Also discrepancies would be relatively more likely at any sites where PMP is poorly estimated.

There are broadly two ways in which FEH and PMP might be combined. One would be by taking some some suitably integrated approach; for a rational basis for this some information on the relative precisions of the two procedures would be needed and a way of eliminating any systematic difference between them. A superficially simpler approach would be to adopt one, say FEH or modification thereof, as standard, but in critical cases to calculate PMP and to give special consideration to any instances where the discrepancy was more than some tolerance level. The latter approach would be the more appropriate if further analysis showed that one approach was appreciably more precise than the other.

It has been assumed throughout this discussion that the PMP estimate used is the physically based one, not the so-called statistical estimate, and also that the physical basis of PMP is sound.

Recommendation 4.1. A further analysis of the relation between FEH and PMP should be made paying attention to the selection of sites for analysis, to explaining the apparent systematic difference between FEH and PMP and the dependence of the ratio on explanatory features such as elevation. Extreme discrepancies should in particular be examined

Recommendation 4.2. The appropriateness and feasibility should be examined of a combined approach involving both PMP and FEH procedures. Some comments in more detail are in the Appendix.

5 The FEH method

5.1 General comments

The FEH method is based on a long and impressive set of investigations. There are in many of the steps of the procedure alternative approaches, slightly or not so slightly different from those used. It is impossible to say without going over the material from the beginning whether an alternative approach would have been better or worse, or, more likely, would have made little material difference to the final answers. I have, where-ever possible checked from the literature or by personal enquiry what tests of the validity of the procedures were employed. The key points are discussed below.

The emphasis in the initial period of work on FEH was on relatively short return periods of a few hundreds of years, later extended to 2×10^3 yr. I understand that there was interest in extending the method to 10^4 yr and this accounts for the software allowing such extrapolation. Funding to investigate the problems in such extrapolation could not be obtained and FEH in effect discourages it. It seems clear that the method in its present form was never intended to be used at such levels.

Very impressive software has been developed to implement the FEH procedures. I understand that to a limited extent it could be updated by amending the values of certain controlling parameters, but that other changes, should such be thought needed, would be expensive to make and perhaps hard to justify so soon after the propagation of FEH. This suggests that if the 10^4 values need changing, it should largely be by providing a correction formula to the present values. There is, however, the additional important point that the rain-gauge data used in FEH was from gauges with at least 9 yr of data up to 1999. Another 5 or more years of data on the relatively new gauges will reduce somewhat the need for spatial grouping and improve precision of, in particular, the index values so that revision to absorb this new data will be desirable at some point and may have an appreciable effect on the extrapolation.

The procedure essentially depends on three stages, the estimation of an index level at a 1 km square grid followed by the formation of a site specific growth curve leading to an estimate by multiplication of the growth curve. This is done separately for each of a range of storm durations. Finally for each site these values are smoothed to produce a DDF plot showing curves of rainfall versus duration at a number of return periods, i.e. probability levels. The main features of these steps are discussed below.

Recommendations associated with these remarks are collected in Section 5. 5.

5.2 Index level

The index level is defined as the median annual maximum rainfall (at a particular duration). It has been produced for a 1 km square grid by a combination of regression analysis to allow for systematic features such as elevation plus spatial smoothing of the residuals by so-called kriging (generalized least squares with estimated spatial dependence). The main limitations of what was done are that the spatial dependence might itself depend on explanatory variables such as elevation, geographical position and so on, and that the calculations are done separately at each duration. That is, the merging of information at different durations is left to a final stage of smoothing. So far as I ascertain while some checks of the validity of the procedure gave reassuring results the possibility of varying spatial dependence was not checked. It is hard to assess how important that might be or whether some preliminary stage of smoothing would have given improved results.

D.A. Jones (unpublished) has shown theoretically that for short series there is an implicit bias in using median annual maximum as a basis for inferring extreme events, leading to estimated rainfalls somewhat too high. It is, however, unclear what the implications of this analysis are when the index level is derived indirectly as in the FEH analysis.

The precision of the index level is shown in map form in FEH (vol 2) for 1 hr and 1 day durations; the areas of high rainfall have highest estimation error. It is not clear from the maps how close to proportionality that relation is. The relation is relevant in assessing the contribution of error in the index level to error in the extreme levels.

5.3 Estimation of growth curves

This is a delicate part of the procedure and an ingenious method, FORGEX, has been adopted of pooling information from more and more sites as one moves into the tail of the distribution. Predictions at a relatively high level of probability are based on sites close the site in question; those at the levels of interest here are dependent on pooling information within a circle of 200km radius. The final answer inevitably depends on a number of assumptions, whose criticality it is impossible to judge without further detailed analysis. Some of the key points are as follows:

- the upper end of the curve is likely to be quite dependent on gauges with long records which may be relatively remote from the site in question
- in particular it is assumed that the whole systematic effect of features like elevation is absorbed in the index value, i.e. there is no consideration of whether the shape of the curve depends on such features

- the effect of spatial dependence is represented by a simple correction assumed to apply in the same way at all probability levels and to be independent of elevation, etc.

- the same index level is assumed suitable for all probability levels.

In this method some extreme events occur more than once at different plotting positions. So far as one can see from published examples the positions do not vary greatly suggesting that the sensitivity to corrections for spatial dependence may not be too large.

5.4 The DDF plot

The final step is to induce appreciable smoothness into the previous calculations, these having been done independently at each storm duration, and at the same time to resolve some logical anomalies. Some such smoothing is essential. In the present case the relations are assumed piece-wise linear on the scale of log rainfall versus log duration.

Again all systematic effects of features like elevation are assumed absorbed in the index level.

As noted by FEH (FEH, vol 2, p.49) this implicitly attaches exponential curves to the FORGEX plots in the second phase and this is critical in the extrapolation to 10^4 yr. This point has been emphasized and criticized by Macdonald and Scott (2001). It is an aspect that should be studied further and indeed tends to suggest that the computed FEH 10^4 levels may be too high; alternative linear extrapolation of the FORGEX plots, as used internally in the FORGEX method, seems at least as plausible and the resolution of discrepancies with PMP would suggest an even flatter relation.

There are other considerations, none more than general suggestions, pointing to the FEH 10^4 levels being too high. One is the comparison with historical events (section 3.1); another is the possibility that the correction for spatial dependence overcorrects the extreme events (section 5.3); a third is the extrapolation issue just mentioned.

5.5 Recommendations

Recommendation 5.1A. The contributions from the various steps in the FEH procedure to the uncertainty in the final answer should be assessed in a few typical cases.

Recommendation 5.1B. The possible effects of features such as elevation, etc. beyond those absorbed in the index level should be analysed.

Recommendation 5.1C. The nature of spatial dependence at high rainfall levels and its effect on the plotting positions used in FORGEX should be reviewed.

Recommendation 5.1D. The precise mode of extrapolation to high levels should be reexamined.

Recommendation 5.2. Rain gauges closest to a sample of Category A and B reservoirs and with more than, say, 25 yr of data should be examined to compare the FEH predictions of maxima with those actually encountered.

There is broad agreement between the recommendations in the present Report and the six primary recommendations of the Babtie group (DETR, 2000). Babtie B1 specifically addresses the DDF curves in FEH; this is a component of the assessment of FEH recommended here in 5. Babtie's B2 and B3 are components of the present 5. Babtie's recommendation B5 is contained within the present 4.2 and B6 is close to 2.1. This leaves B4 about data. Section 5.1 of the present Report notes the existence of several more years of data since FEH was written and that this is particularly important for the gauges where only the minimal number of years of data were available. No specific recommendation was made on this point but if and when FEH is revised this data should be used in line with B4.

6 Final assessment

In summary brief answers to the originating questions are as follows:

- Are the results generated by the FEH software to derive 10,000 yr return period rainfall, by extrapolation of relationships held to be valid up to 2,000 yr return periods, valid? Is the methodology technically defensible?

  The method is in principle well thought out and based on very extensive study of the relevant data. It was, however, neither designed nor intended for use at such extreme levels. It is therefore likely that modifications are needed for the extreme rainfalls but hopefully these can be accommodated within the broad FEH approach. There is some limited largely indirect evidence, which may turn out misleading, that the 10,000 yr predictions are too large.

- If not, can the method be improved to cover this range of return periods?

  Further study is needed as to the nature and extent of improvements, if any, that are desirable.

- If it can be improved, what is needed in the way of revised analysis?

  These are set out in some detail above.

- If it cannot be improved what options are there for derivation of such extreme rainfalls?

  This does not arise.

- If the method is valid and meaningful, how do we account for the contradiction with PMP?

  There is probably no contradiction in the light of realistic assessments as to the uncertainties in PMP and the FEH values. The issue is rather that of a hydrological-meteorological judgement as to whether PMP uses important information not implicitly employed in FEH and, if it does, as to how and when a combination of FEH and PMP values should be employed.

D.R.Cox
Nuffield College, Oxford

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APPENDIX

The object of this Appendix is to set out some statistical points in a very preliminary form with the hope of helping any further work that may be done on these issues.

The attempts to attach a return period, i.e. probability of exceedance, to PMP have given very variable results. The methods used are not set out at all explicitly in the literature and in particular WMO (1986) is totally unhelpful. It seems best to begin by considering the simpler, but still difficult, question: what is the precision of the estimated PMP? This is best tackled by estimating the standard error of $P = \log$ PMP; thus a standard error of about 0.10 in P would mean that roughly 95% of the time the estimated PMP is within $\pm$ 20% of the notional true PMP.

To estimate this standard error, which will depend on site and duration, in principle one should identify a number of meteorological and other features, as independent as feasible, that essentially determine PMP. Call these $U_1, \ldots, U_q$ and assess the best possible estimates of their measurement errors by giving standard errors $\sigma_{U1}, \ldots, \sigma_{Uq}$. Compute P at values of $U_1 \pm \sigma_{U1}$, etc and call the difference between the two values $2\Delta_1$, etc. Then the standard error σ_P of $P = \log$ PMP is approximately

$$\sqrt{(\Delta_1^2 + \ldots + \Delta_q^2)}. \tag{1}$$

(There are many variants of this; for example the differences can be found via partial derivatives if explicit formulae are involved.) Note also that the relative magnitudes of the Δ's indicate which are the critical components in the process of determining P.

That process is not easy but surely much easier than determining a return period.

Now in principle a similar procedure can be used to attach a standard error σ_F to the log FEH value at a given site, a given duration and at a given probability level. It will depend on errors in the index level and on errors in the FORGEX curve at the probability level involved. Note that this takes no direct account of the smoothing inherent in deriving DDF curves.

Some assessment, however crude, of the relation between σ_P and σ_F seems crucial to decisions on how, if at all, to combine them. To use PMP at relatively high probability levels, e.g. 0.05 or 0.01, would require a correction factor and unless that were virtually error free another component would have to be added to σ_P^2. Even without that, it seems likely that there will be three regimes of probability of exceedance: those in which FEH is clearly preferable, an intermediate zone and one where, under the assumption that there is a real upper limit, estimated with error, PMP or some minor amendment thereof, will be important and maybe dominant.

To study the behaviour of PMP at low probability levels (long return periods) and in particular to attach a return period to the measured value of PMP one approach in outline would be as follows. Greek letters denote unobserved quantities.

Suppose that at a given site and duration there is an absolute maximum log rainfall, Π, say. Suppose that Π is estimated by P, normally distributed with mean Π and standard deviation σ_P, estimated as above. Suppose further that for rainfall levels close to the absolute maximum

$$\text{prob}(Y > \Pi - t\sigma_P) = \alpha t^{\beta} \tag{2}$$

for small positive $t\sigma_P$; for negative t the probability is zero. The quantity of interest is $\text{prob}(Y > P - t\sigma_P)$, i.e. the same probability using the observed P rather than the true Π. Over a set of applications Π itself has a distribution, which we assume for the present widely dispersed. Then it can be shown that approximately

$$\text{prob}(Y > P - t\sigma_P) = \alpha \int_0^\infty z^\beta \phi(z - t) dz, \tag{3}$$

where

$$\phi(v) = \exp(-v^2/2)/\surd(2\pi). \tag{4}$$

This is easily computed as a function of t and β using the functions tabulated by Abramowitz and Stegun (1965, Chapter 26).

In particular if $t = 0$, then the probability is the reciprocal of the return period at the estimated PMP itself, and is

$$2^{\beta/2-1}\pi^{-1/2}\alpha\Gamma\{(\beta + 1)/2\}, \tag{5}$$

where $\Gamma(.)$ is a standard function (Abramowitz and Stegun, 1965, Chapter 6). This is obtained by setting $t = 0$ in equation (3) and transforming the resulting integral by writing $t^2/2$ equal to a new variable of integration, v, say.

If in particular $\beta = 0, 1, 2$, although at the moment I cannot see any *a priori* argument for choosing β, then the return period of PMP would be

$$2/\alpha, \quad \surd(2\pi)/\alpha, \quad 2\surd 2/\alpha \tag{6}$$

and thus its approximate estimation would hinge primarily on estimating α with relative insensitivity to β at least in this range. This might be possible either from empirical data on extremes or by studying the relation between FEH and PMP values, assuming the former relatively reliable at modest levels.

The general form of relation (3) emerging from this is in line with the suggestions made by Mr Alan Brown (KBR) about the relationships to be expected at high levels; as it approaches the PMP the rainfall versus probability curve flattens and increases only slowly above the observed PMP, although there is no strict upper limit. The Table illustrates what happens with $\alpha = 2 \times 10^m, \beta = 0, \sigma_P = 0.10$. That is, the return period at PMP is assumed to be 10^m yr, where perhaps $m = 5, 6$, and the estimation error in PMP has a standard deviation of 10 %. Changes in α, σ_P rescale the relation but do not change its form. See also the Figure.

Table. Hypothetical special case. Relation between rainfall, R; probability level, pr $\times 10^{-m}$, return period, RP, yr $\times 10^m$, near to estimated PMP; $\alpha = 2 \times 10^m, \beta = 0, \sigma_P = 0.10$.

t	log (R/PMP)	R/PMP	pr	RP
2	-0.2	0.82	1.95	0.51
1	-0.1	0.91	1.68	0.59
0	0.0	1.00	1.00	1
-1	0.1	1.11	0.32	3.2
-2	0.2	1.22	0.045	22.0
-3	0.3	1.35	0.0027	370

At about 10% below PMP the return period is more than one-half that at PMP whereas 10% above PMP the return period is over three times that at PMP with more extreme asymmetry at more extreme levels relative to PMP, although the values at high positive values of t have to be interpreted very cautiously as they depend strongly on the assumption that $\beta = 0$; this is why the Table has not been extended beyond $t = 2$.

As noted in the main Report there are various ways in which information from FEH and PMP could be combined. One aspect, in effect suggested by Mr Brown, would be to use some development of the above arguments to guide the extrapolation of the FEH method to the extreme levels which, as noted, the FEH method in its present form was not designed to address; the outcome would be a method synthesizing FEH and PMP. While the formulae above can all be evaluated via functions tabulated by Abramowitz and Stegun (1965, Chapters 6, 26) it is to be stressed that the outline analysis given here is highly preliminary and is set out as a possible basis for further development not as material for immediate application.

Figure. Hypothetical special case. Plot of Rainfall magnitude/PMP against probability level (or equivalently return period). 10% standard deviation in determining PMP. β = 0. If unit of probability is one per 10m dam years, return period is in years times 10m.