Developing an insight into the particle-scale mechanisms that underlie suffusion in granular filters

FREYA SUMMERSGILL, Imperial College London, UK
CATHERINE O'SULLIVAN, Imperial College London, UK

SYNOPSIS. Suffusion is a potential mechanism for internal erosion in the filters of dams and embankments. The mechanisms that drive the initiation and subsequent propagation of suffusion operate at the scale of the individual particles. Consequently it is difficult to analyze these mechanisms in detail using conventional experimental or numerical techniques. Discrete element modelling (DEM) is a method of numerical simulation that explicitly considers individual particles, their motions and the forces that are generated between them. This paper discusses the use of a 2D DEM model to analyze the influence of the particle size distribution on the material microstructure. Specifically the variation in contact forces, particle stresses, void ratios, pore size distribution and the connectivity of the particles are considered. While the results of these 2D simulations cannot be directly applied to real 3D soils, insight to inform our understanding of the mechanisms is gained.

INTRODUCTION

The term suffusion refers to the ability of the finer particles in a dam to move or diffuse through the coarser particles. Earlier research studies have recognised that this mechanism operates at the scale of the individual particles, however detailed analysis has been restricted because the soil particles are three-dimensional, opaque and very small. While the particle scale mechanics can be inferred from observations of overall response in the laboratory, micro-scale data to inform a scientific understanding of the initiation and propagation of suffusion is lacking. Discrete element models (DEM) are emerging as a popular tool in the geomechanics research community to analyze the micromechanics of soil response. Discrete element simulations can be carried out in either two or three dimensions to model individual particles and their interactions. While real soil is three-dimensional and any quantitative conclusions about suffusion would require a 3D study, a two-dimensional study is useful as the micro-parameters can be directly visualized, and so a 2D study forms the basis for a future, more in-depth study. This paper describes a preliminary 2D study of two "virtual" samples created with differing particle size distributions. The analysis of their microstructure allows an assessment of their relative susceptibility to suffusion.

SUFFUSION AND THE PARTICULATE NATURE OF SOIL

Suffusion is a form of internal erosion in which mass erosion of the soil or fill occurs due to seepage flow through an internally unstable soil (Brown, 2007). Internal instability refers to the inability of a soil to act as a filter to prevent loss of its own particles (Kenney & Lau, 1985). The high seepage flows present in hydraulic structures such as dykes and embankment dams can increase susceptibility to suffusion which is particularly problematic in these situations because a change in the soil or fill properties can be very dangerous (Bonelli et al., 2006). Removal of material from within the foundation of a dam will create a more open soil structure. This leads to increased permeability and seepage, which can cause progressive deterioration of the foundation, an increased possibility of settlement of the embankment and a higher risk of toe instability. The migration of the particles from a filter renders it coarser and less effective in protecting the core materials from erosion, leading to an unintentional loss of water from the reservoir or even wash out and failure of the dam core (Wan & Fell, 2007).

While the impacts of suffusion on embankments can be observed at a macro scale, the mechanisms and soil properties that influence the process operate at the scale of individual particles and they are governed by the interaction of the particles at this level. The features of soil that influence this interaction will control the internal stability of the soil. The current theories relating to the mechanisms of suffusion are explained in terms of the loose particles, the size of constrictions between particles, the potential travel distance of particles, perfect filters and stress distribution. Guidelines have been developed to assess the internal instability of soil using these theories and laboratory experiments that relate the particle size distribution (psd) curve to the percentage loss of fines under high hydraulic pressures (Kenney & Lau, 1985), (Wan & Fell, 2007), (Skempton & Brogan, 1994).

Considering soil as a particulate material at the micro scale, Kenney and Lau (1984) described a suffusive soil as having the following properties:

(i) A primary fabric of individual particles which support loads and transfer stresses.

(ii) Within the pores of this primary fabric there can exist particles that are not fixed in position and do not transfer effective stresses.

(iii) Under the pressure of seepage these latter particles are movable within the pores and can be moved into neighbouring pores if sufficiently small. Some of these loose particles can be transported by seepage flow through the matrix of load bearing particles and out of the soil.

Materials composed of uniform-sized particles are stable independent of their density and the severity of seepage (Kenney & Lau, 1985) as the pore constrictions between particles are of a similar size which is smaller than the particles themselves.

Skempton and Brogan (1994) suggested that the mechanism of suffusion is slightly more complex than loose and fixed particles. They proposed that particles carrying a (small) proportion, α, of the effective stress are those most likely to move and that suffusion initiates when the effective stress in these particles becomes zero. Their tests indicated that suffusion can occur at hydraulic gradients that are far smaller than the theoretical critical value of 1 calculated for a homogenous granular material.

It is clear that suffusion as a potential hazard to embankment dams and foundations merits thorough explanation. To develop a scientific understanding of suffusion at the particle scale use of either advanced experimental techniques such as micro-scale computed tomography or numerical simulations that simulate the particle interactions is required. The current study explores the use of the numerical DEM to develop insight into suffusion and evaluate the presence of loose and fixed particles and their stress.

THE DISCRETE ELEMENT METHOD (DEM)

The Discrete Element Method (DEM) was originally proposed by Cundall and Strack in 1979. Conventionally in numerical modelling of soil the material is assumed to be a continuum, however in DEM the individual particles and their interactions are modelled. A key assumption in DEM is that the particles themselves are rigid; however a small amount of overlap is allowed where particles come into contact, as illustrated in Figure 1. This overlap is analogous to the deformation that occurs at the contact between real soil particles and is used to calculate the compressive force transmitted between two contacting particles. Simple Coulomb friction is used to simulate the sliding of particles around each other. DEM simulations are a dynamic analysis, where the dynamic equilibrium equation is solved for each particle at discrete time intervals to calculate the particles' acceleration. These acceleration values are used to calculate the particle motions using a central difference type time integration. The particles themselves are restricted to geometries that can be analytically described, with the most common particle shapes being discs (2D) or spheres (3D).

Prior studies have shown that this approach can replicate the key features of soil response. DEM simulations can be carried out in either two or three dimensions. Both two and three dimensional DEM simulations are highly computationally expensive as a consequence of the large number of particles and contacts involved in even analysis of a relatively simple problem. The sequence of calculations is illustrated in Figure 2. The simulations are dynamic or transient and the system is non-linear as the contact state of the particles evolves during the simulations. From the perspective of suffusion these simulations can provide important information including the movements of individual particles and the number and magnitudes of the contact forces acting on the particles.

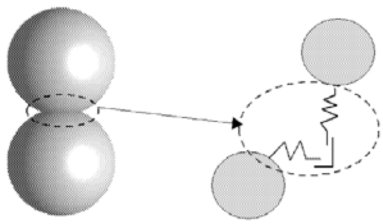


Figure 1. DEM modelling of particle contacts

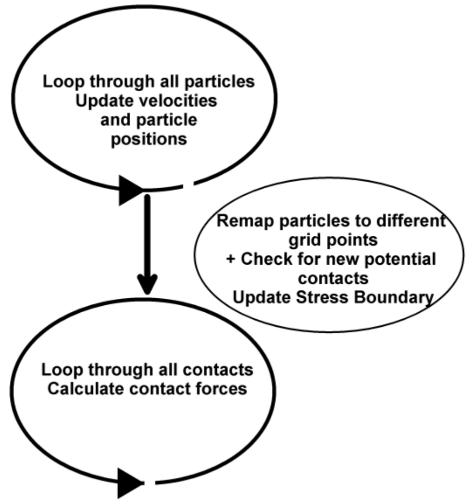


Figure 2. DEM force calculation cycle

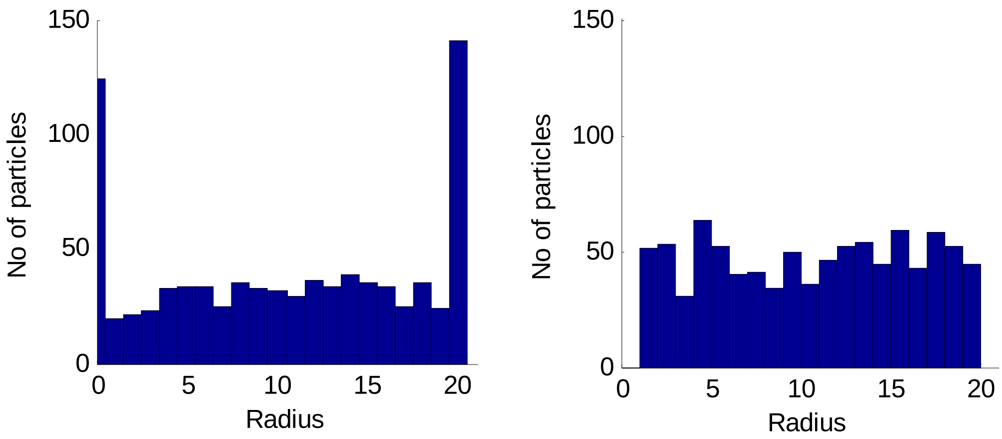
As suffusion is a 3D problem that involves the movement of particles through a mass of soil under the action of water flow, a highly complex coupled 3D model would be required for a quantitative investigation. An in depth study would include simulation of the fluid phase of the model by coupling DEM with computational fluid mechanics. The current study is not coupled; rather it is a preliminary 2D parametric study that allows a qualitative assessment of the important micro-scale parameters, from which useful insight in to the mechanism can be achieved.

2D DEM STUDY

Methodology

In the current study "virtual" 2 dimensional (2D) samples of soil were created using the commercially available particulate DEM program PFC2D, Itasca. The samples were created to assess the effects of various parameters, including particle size distribution on the void ratio, interparticle contact forces, and particle stresses. Here consideration is restricted to two representative simulations; for further details of the study refer to Summersgill (2009). Both samples considered have a range of particle sizes between 1 and 20 and interparticle friction was set to 0. The samples differ in the distribution of sizes with both Gaussian and Uniform distributions being considered. These distributions were selected to explore the hypothesis of Kenney and Lau (1985) that soils with a more uniform particle size distribution have a lower susceptibility to suffusion.

For both samples the arrangement of 2D particles or discs was created using a random number generator. First a radius was selected using the chosen distribution and two radii values defining the range. The distribution of radii produced for each sample is illustrated in Figure 3. The number of discs generated for each distribution was similar so the graphs are comparable. The Gaussian distribution generates the disc radii using the Gaussian curve and the range of radii is considered two standard deviations apart on the curve so 68.2% of the discs will have a radius within this given range. Figure 3(a) shows that a smaller proportion of the discs have radii between 1 and 20 compared to the Uniform Distribution (Figure 3(b)) but larger and smaller discs were also generated. The Uniform distribution only generates particles sizes within the range of radii, 1 to 20.



(a) Gaussian Distribution (b) Uniform Distribution

Figure 3. Histograms of the size of particles produced

The resultant Particle Size Distribution (PSD) curves produced for the two distributions used in these simulations are shown in Figure 4. The simulations underwent a different number of expansion cycles so the range of disc sizes differed and normalised radii values are plotted to facilitate direct comparison. The normalised radius is calculated by dividing each radius by the mean radius for the relevant specimen. The Gaussian distribution produces an S-shaped PSD curve typical of many soils. The tail and head of the S shape indicate the presence of small and large particles. The Uniform distribution produces the parabolic shaped PSD curve which indicates the absence of very small or very large particles. The range of particle sizes present in each sample can also be compared in Figure 6(c) and (d) along with their locations at the end of the simulation.

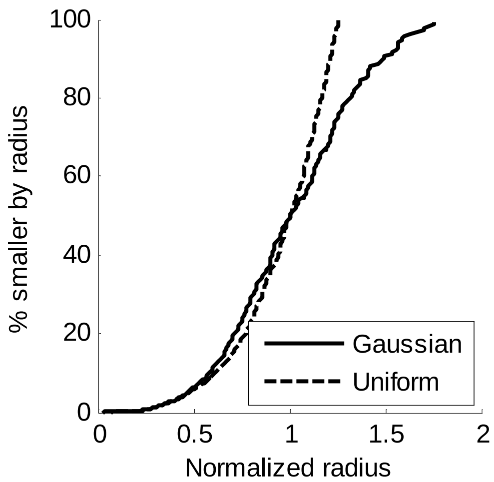


Figure 4. PSD Curves for Gaussian and Uniform Distributions

The specimen generation was carried out by adding new discs to the system sequentially. The centre of each new disc was then placed in a randomly selected location inside the four designated walls so that it did not overlap with previously placed discs or walls. The insertion process was continued until a specified number of discs (in this case 1000) were created. The resulting sample then contained an assembly of non-contacting discs. The next stage of the analysis was to introduce contact between the particles by gradually increasing the particle sizes. The gradual expansion was achieved by multiplying the radius of every particle by a coefficient, α, with α>1. Expansion of the particles generated inter-particle forces and the particles were allowed to move and adjust their positions following the calculation cycle shown in Figure 2 until the system came into a state of static equilibrium. This approach generated dense samples with an isotropic distribution of stresses and contact forces. During the expansion the value α was determined by considering the coordination number, N.

$$N = \frac{2N_c}{N_p}$$

where N_c is the number of contacts and N_p is the number of discs (or particles). When N<2 the sample cannot transmit stress (it is not “percolating”) and the particles are rapidly expanded, with the expansion rate significantly decreasing once N>2.

<u>Results</u>

In laboratory experiments that investigate suffusion it is common to use the voids ratio, e, and particle size distribution curves to characterize samples.

These parameters can also be considered in the DEM model, along with particle scale parameters of radius, contacts and stress.

Contacts

Consideration of the density of contacts in the system may give insight into suffusion susceptibility. Figure 5 illustrates the evolution of the average number of contacts or coordination number (N) and void radio (e) as the radius expansion progressed. As would be expected the void ratio decreased as the contact density increased, however the relationship between N and e is non-linear. In fact while the Uniform distribution sample gives a higher N than the Gaussian distribution both samples reach similar values of void ratio. It should be noted that the e values are those for a 2D sample of discs and cannot be directly related to a real 3D soil. The very high e at the start of each simulation is due to the initially loose nature of the sample before the discs are expanded. The very low e values are due to the overlap of particles during the simulation which is necessary for the modelling of contact forces. The Gaussian distribution with this range of disc radii can produce a comparably dense sample with fewer contacts between particles.

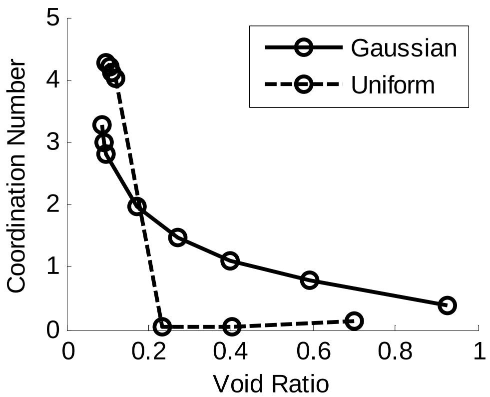
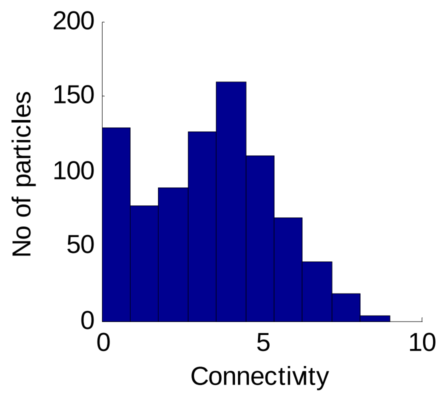


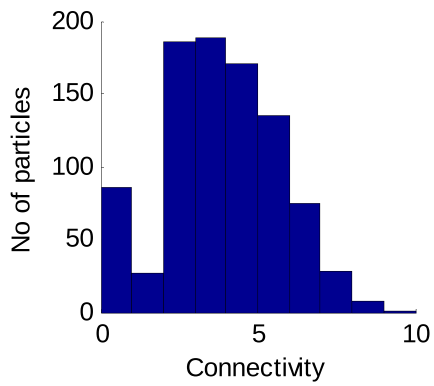
Figure 5. Variation of Void Ratio with Coordination Number for both distributions

The individual number of contacts per particle was also monitored; in network analysis terminology this gives the degree distribution. The degree is the number of contacts a particle participates in. The degree distribution for the two samples considered here is illustrated in Figures 6 and 7. Figure 6 is histogram of the number of contacts considering all particles in the system, while Figure 7 illustrates the variation in the degree as a function of particle size. Clearly there are a greater number of particles with two or fewer contacts in the Gaussian sample in comparison with the Uniform sample. This supports Kenny & Lau, 1985 that a uniform psd creates a more stable material. The histograms in Figure 6 also show that the discs of the Gaussian distribution sample have a wide variability in the number of contacts but the majority of discs for the Uniform sample, Figure 6(b), have three, four or five contacts.

Considering the relationship between particle size and connectivity it is clear from Figure 7 that for both distributions the smaller discs have a higher likelihood of having 2 or less contacts. (normalized radius =1) have zero contacts. There are some particles with a greater than average size in the Gaussian distribution that are metastable with one or two contacts but almost no particles sized above average in the Uniform distribution are metastable.

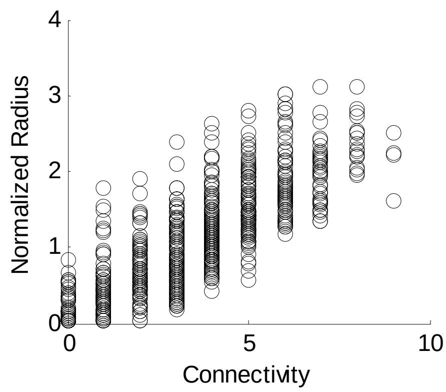


(a) Gaussian Distribution

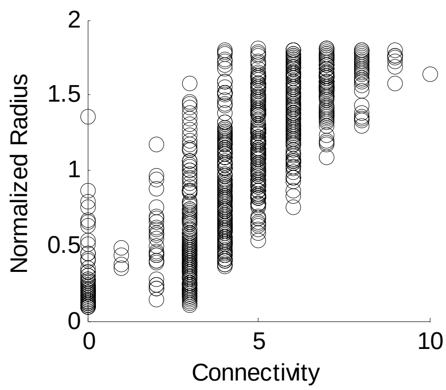


(b) Uniform Distribution

Figure 6. Histogram of the number of contacts for each disc



(a) Gaussian distribution



(b) Uniform Distribution

Figure 7. Variation of particle connectivity with radius

A coordination number of 2 was noted as a bound to stress percolation above. The reason for emphasizing a coordination number of 2 as a stability boundary for suffusion considerations can be appreciated by reference to Figure 8(a). In this Figure particle ‘b’ is in contact with a particle ‘c’ above and particle ‘a’ below. The major principal stress is orientated in the vertical direction and the system is subject to horizontal flow. From a mechanical perspective this configuration is metastable or potentially unstable. Considering only the stresses acting on the system, an increment of vertical stress is likely to cause the contact chain a-b-c to experience a buckling type failure. At each contact point a normal and shear force will be transmitted and the shear force will be substantially smaller than the normal force. It is only the particle inertia and horizontal component of the shear force that will provide a resistance to the drag force imparted by the horizontal flow. Particle 'b' is highly susceptible to suffusion.

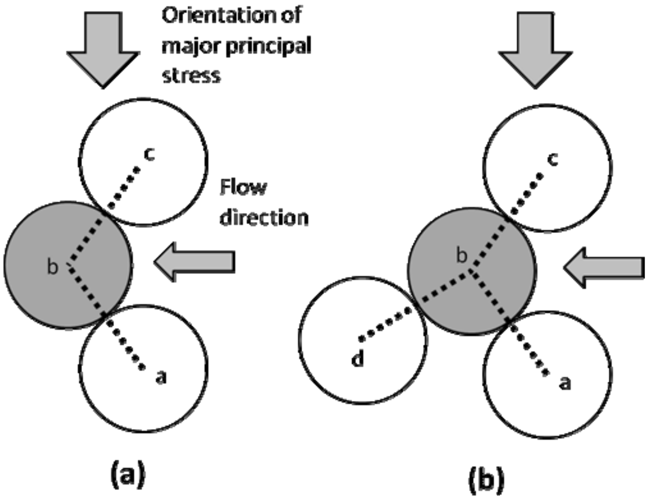


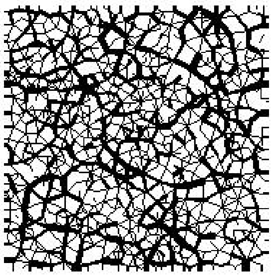
Figure 8. Mechanical and Hydromechanial Stability of Particles

Consider now Figure 4(b), in this case the addition of particle 'd' renders the system stable with regard to an increment in vertical stress. Furthermore the horizontal component of the normal contact force imparted by particle 'd' on particle 'b' will now contribute to resist the flow induced horizontal drag force acting on particle 'b'. Thus the risk of suffusion is now greatly diminished. While the suffusion susceptibility of a given particle will depend on the specific geometry of the contacts that the particle participates in, there clearly is a transition once the coordination number increases above 2. The concept of a requiring a minimum coordination number of 3 for mechanical stability (i.e. stability under increments in the principal stresses) is well established amongst the granular mechanics community.

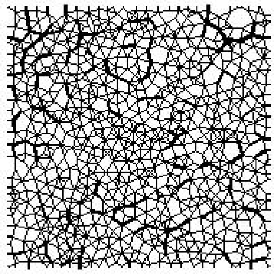
Further insight into the internal structure of the material can be achieved by visual observation of the contact force network (Figure 9). The contacts are represented by lines joining the centres of contacting discs which have a thickness corresponding to the size of the force between the two particles. The Uniform distribution with discs of variable size having three, four and five contacts creates an even, homogenous network of contacts, as shown in Figure 9(b). The Gaussian distribution, Figure 9(a), has a more heterogeneous contact force network. The size of particles involved in the each node of the contact force network can be inferred from Figure 9 (c) & (d).

Stress Distribution

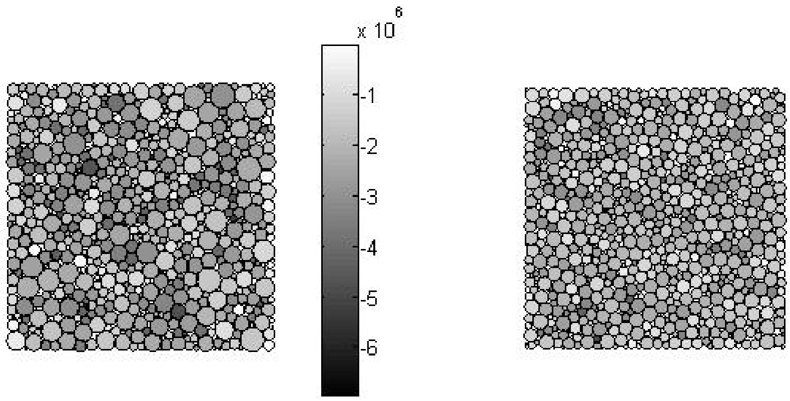
Figure 9 also gives insight into the average stresses within the particles. The darker the colour the greater the average particle stress. One would anticipate that the most highly stressed particles are those less likely to move under the action of seepage force. In both simulations it is not only the larger discs that form a primary network of load bearing particles. Smaller discs are also included in the network of forces, as clearly indicated by the presence of small highly stressed particles. The discs in the Uniform distribution sample are more evenly stressed than in the Gaussian distribution sample. The Gaussian distribution plot, Figure 9(c), shows a greater number of dark highly stressed discs and light weakly stressed discs.



(a) Gaussian distribution: Contact force network at end of simulation



(b) Uniform Distribution: Contact force network at end of simulation



(c) Gaussian distribution: Particle stress and location at the end of the simulation.

(d) Uniform distribution: Particle stress and location at the end of simulation.

Figure 9. Particle stresses and contact force networks

The relationship between the stress felt by a particle and its size is complex as illustrated by the individual particle data plotted in Figure 10. This is a plot for the Uniform distribution but shows the same absence of a clear relationship as the Gaussian distribution. The normalized stress is used to emphasis the spread of data from the mean value. The results could assist the proposal by Skempton & Brogan that the stress on a particle is important in the initiation of suffusion. A extension of the current simulation would be required to test this theory by applying a body force to the 'sample' produced at the end of simulation and identifying the particles which are moved.

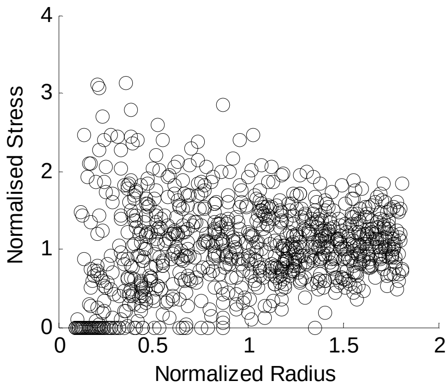
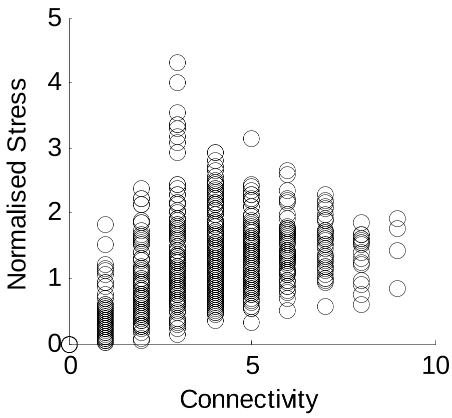
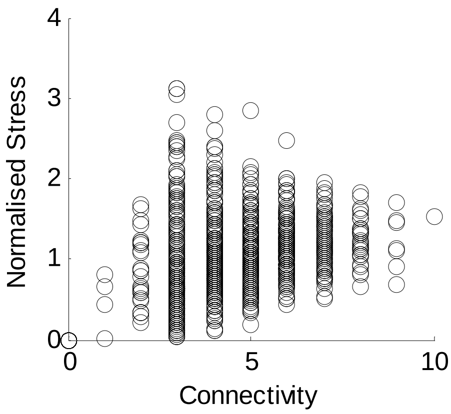


Figure 10. Stress and Radius for each disc

There is a correlation between connectivity and stress. In both distributions the particles with three contacts experienced the widest range of stresses. With an increase in the number of contacts, the range of stresses felt by the particles decreases and converges towards the mean value, as illustrated in Figure 11.



(a) Gaussian distribution



(b) Uniform distribution

Figure 11. Stress and number of contacts for each disc

CONCLUSION

Suffusion is a process that has implications for dam safety. The underlying physical mechanisms operate at the scale of the individual particles, and, until relatively recently, their analysis has been intractable. Discrete element modelling is a numerical modelling technique that makes simplifying assumptions about the particle geometries and their interactions to facilitate simulation of the particle movements, interactions and can gain information about the inter-particle forces. Ultimately, quantitative micro-scale data on suffusion can only be achieved using a coupled fluid-particle, 3D DEM code. However, this paper has shown the results of a preliminary study that has demonstrated that useful insight into the mechanisms can be achieved in relatively computationally simple 2D decoupled analyses.

Here a comparison of a the microstructure sample with a Gaussian distribution of particle sizes and a sample with a Uniform distribution of sizes allowed some interesting conclusions to be made, as summarised here:

(a) In two dimensions a particle which is participating in two or fewer contacts is either in an unstable or metastable state and will have little resistance to movement under the action of a fluid drag force.
(b) There are more metastable or unstable contacts in the Gaussian specimen (with a wide range of particle sizes) in comparison with the Uniform specimen (with a narrow distribution of particle sizes).
(c) In the Gaussian specimen some particles with radii that are greater than the average radius are metastable; in the Uniform sample almost all particles bigger than the average particle are stable.
(d) The relationship between particle mean stress and particle size is complex, and there are a number of small highly stressed particles.
(e) The particles with a larger number of contacts are likely to experience the mean stress.
(f) The particles with three contacts that just meet the lower boundary for stability also experience the largest range of stresses.

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