

Assessing the vulnerability of a typical British embankment dam to internal erosion

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SYNOPSIS The vulnerability to internal erosion of a 100-year old typical British embankment dam without filters has been examined using seven methods. Four assessed the potential of the shoulder fill, a low permeability till, to act as a filter and prevent erosion through the core. These were the Sherard grading criteria, the Vaughan permeability criteria, the Delgado permeability criteria, and the Foster & Fell probability-based criterion. Two methods, particularly Fry's, assessed whether sufficient tractive forces would be available to initiate and continue erosion. The seventh was the Interim Guide to Quantitative Risk Assessment for UK Reservoirs, which by comparing the characteristics and performance of the subject dam to that of similar embankment dams makes it possible to assess the probability of failure and whether this is acceptably low. The filter and probability criteria gave only moderately re-assuring results, probably because they are derived from stringent tests to develop conservative design criteria for filters. Methods, such as Fry's, that analyse the mechanics of erosion in the dam seem to best reflect the actual performance of the dam, which has been demonstrated by the QRA to be good when compared to the behavior of many similar dams.

SAFETY INVESTIGATIONS

A 2003 dam safety report recommended safety investigations of the vulnerability of a typical British embankment dam to internal erosion. This was not because there was any obvious cause for concern, but because at the time there was a renewed appreciation of the possibility that internal erosion may pose an unacceptably high threat to the integrity of such dams and their vulnerability to it was not being systematically examined. Such dams do not have filters to protect the core from erosion, and the investigations therefore examined the permeability and grading of the shoulder fill and foundations and the properties of the clay core of the dam with the objective of checking the capacity of the fill to act as a filter and prevent erosion of the core, and the erosive capability of seepage water flowing through fill into and out of cracks in the core, making use of the following:

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- Vaughan and Soares (1982) 'perfect' filter relationship between a filter's permeability and its filtering capacity
- Sherard and Dunnigan's (1989) 'critical' filter relationship between a filter's grading and its filtering capacity.
- Delgado et al's (2006) approach linking the percentage of the base soil passing the 0.075 mm sieve to the filter permeability.
- Foster & Fell's (1999, 2001) (approach using logistic regression method in order to replace deterministic no-erosion criteria by a probability-based approach
- Assessments of seepage velocities and forces available to initiate and sustain erosion using Bridle (2007), Fry (2007), Fry & Blais (2006) and Fell & Fry (2007).

This is a work in progress, laboratory investigations have been completed to date, boreholes to check the in-situ properties of the fill have yet to be completed. Many of the findings were reported in the Intermediate Report of the ICOLD European Working Group on internal erosion (Bridle, Delgado and Huber, 2007) and are repeated below.

The Interim Guide to Risk Assessment for UK Reservoirs (Brown & Gosden, 2004), a primary purpose of which is to assess the vulnerability of typical British dams to internal erosion, was published after the 2003 inspection, but an assessment has recently been made and the results are reported below.

THE DAM

The dam was completed in 1904. It is 22.5 m high in earthfill with a puddled clay core 2.7 m wide at the crest with side slopes of 1 to 12 above a clay filled cut-off trench. The upstream slope is 3 to 1 and the downstream slope is 2 to 1. The embankment fill is glacial till.

Photographs (Figures 1, 2) show the puddled clay at the top of the cut-off trench and the site railway that transported the till to the dam embankment, where it was placed on the dam shoulders by side-tipping from the railway wagons. No in-situ tests of the fill's permeability or density have yet been completed, but the photographs indicate that although there is some segregation of the fill, the method of placing may have produced dense fill of varying grading and permeability.

LABORATORY PERMEABILITY TESTING OF GLACIAL TILL SHOULDER FILL

The permeability of the fill in the shoulders of the dam has been established by tests in the large-scale TRL permeameter. This permeameter is 0.3 m square and 1.0 m long. The samples are placed in layers. Water is

BRIDLE

introduced under pressure at one end and flows along layers of fill, as it would from an opening in a core into the adjoining shoulder fill, and flows out at the other end of the permeameter.



Figure 1: Puddled clay fill placed in cut-off trench up to level of dam foundation



Figure 2: Fill placed on shoulder of the dam by side tipping from railway wagons

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The till is stony with many fines (Figure 3). The samples tested necessarily omitted particles larger than 37.5 mm, about 20% of a typical sample, as Figure 4 shows. The in-situ permeability of the fill has not yet been measured, but the permeability of the truncated fill may be representative of much of the fill in-situ, because the in-situ fill, having been side tipped will be variable, and because there is little difference between the full and truncated grading around 10% passing, which is thought to have most influence on permeability. The permeability varies with density as shown in Figure 5.



Figure 3: Sample of glacial till used for shoulder fill

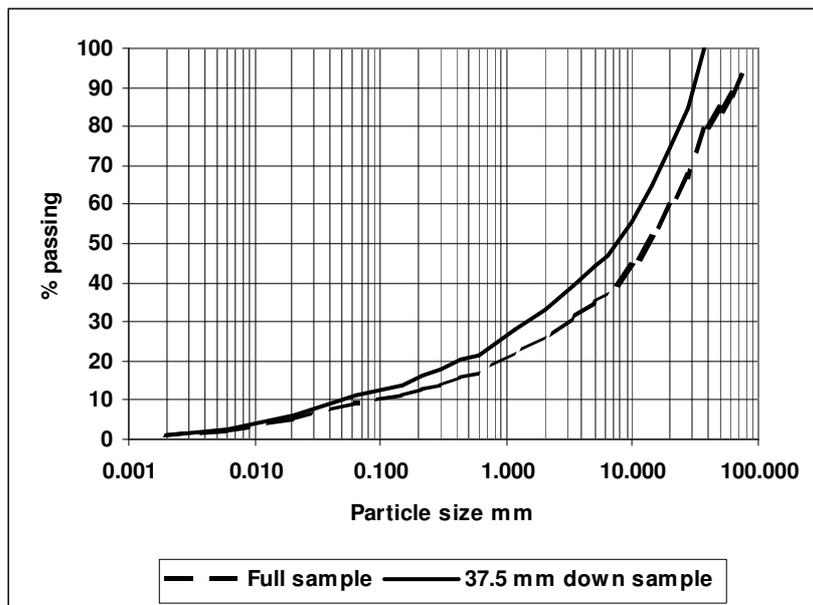


Figure 4: Grading of glacial till used as shoulder fill



Figure 5: Permeability of glacial till fill as measured in TRL permeameter

Vaughan & Soares (1982) derived the following relationship between permeability and filtering capability:

$$\delta_R = 2.54 * 10^3 (k)^{0.658}, \delta_R \text{ in microns, size of floc trapped; } k, \text{ permeability of filter in m/s}$$

Using the relationship, the fill could trap flocs or particles from 0.3 to 4.2 microns, depending on its density

TESTS ON CORE SAMPLES

The grading of the core was determined from conventional dispersed samples and from undispersed samples. The objective was to determine the smallest floc size present in the core because the Vaughan and Soares 'perfect' filter approach requires that there is no movement of the soil into the filter.

The core samples proved to be very 'murky' (Vaughan & Bridle, 2004), making it impossible to assess the smallest floc size present using Stokes' Law tests because the falling front was not visible through the 'murk' and the velocity at which it was falling could not therefore be measured. The 'murk' was investigated particularly by 'sieving' on very fine cellular micro-sieves and glass fibre filters able to retain particles (or flocs) down to 0.45 microns, smaller than the usual minimum of 2 microns (0.002 mm) measured by pipette in standard soil mechanics laboratory tests. The tests

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showed that (at this dam, at least) the ‘murk’ cannot be ignored as colour or other property without physical substance. It was not possible to determine the size of the smallest floc or particle present, but about 7% passed the 0.45 micron ‘sieve’, as the grading curve on Figure 6 shows.



Figure 6: Complete grading of core fill including use of very fine cellular micro sieves and glass fibre filters from 0.063 mm down to 0.00045 mm (0.45microns). Note apparent high clay (0.002 mm) content of dispersed sample (45%), lower clay content (17%), high silt content (83% maximum) of undispersed sample. Liquid limit of core is 35%, plasticity index 19%, on the CL-CI boundary, clay of low to intermediate plasticity.

FILTERING CAPABILITY OF GLACIAL TILL SHOULDER FILL USING VAUGHAN AND SOARES PERMEABILITY CRITERIA

The core may be particularly vulnerable to erosion because, as Figure 6 shows, it may be as much as 83% silt (2 – 60 microns) and therefore erode at a lower seepage velocity than cohesive clay.

The glacial till fill is capable of acting as a filter, trapping, when dense, particles or flocs as small as 0.3 microns. The particle size of the smallest particle present was not determined, but it was proved to be smaller than 0.045 micron and could have been smaller than 0.3 micron, in which case about 7% of the weight of the core could be eroded into the fill, causing some damage. If the fill near the core were less dense, it would trap particles

BRIDLE

or flocs above 4.2 microns equivalent diameter, but as flocs smaller than these comprise somewhere between 8% and 29% of the weight of the core, the damage could be serious.

FILTERING CAPABILITY OF GLACIAL TILL SHOULDER FILL USING SHERARD GRADING CRITERIA

The results of considering the glacial till fill as a filter to the core following 'critical' filter rules (Sherard & Dunnigan, 1989; USDA, 1986) are shown on Figure 7. The fill would be too coarse in the larger sizes and too fine in the smaller sizes to be the Category 2 filter required. However, the important D_{15} size is 0.4 mm, below the recommended maximum of 0.7 mm and above the recommended minimum of 0.1 mm.

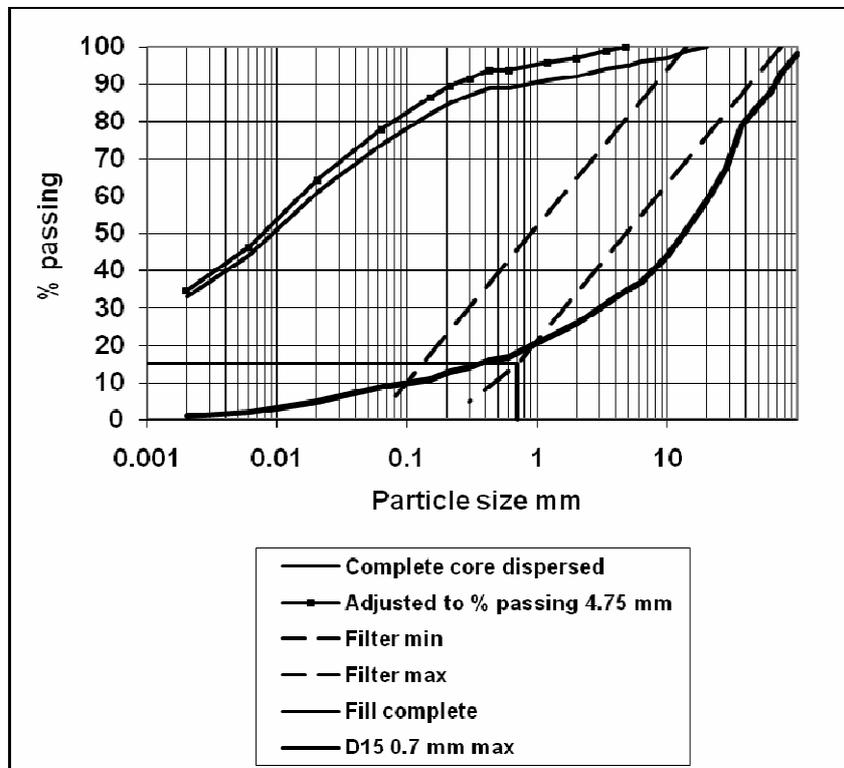


Figure 7: Considering glacial till shoulder fill as 'critical' Category 2 filter to core. Fill too coarse in larger sizes and too fine in smaller sizes, but meets $0.1 \text{ mm} < D_{15} < 0.7 \text{ mm}$ criterion

The dispersibility of the core was assessed using the pinhole test. It was a non-dispersive, ND2, soil. The fill's capability as a filter was checked using the no-erosion filter test, which showed that no erosion occurred and that the sides of the 1.0 mm hole through the till had not been eroded. It could be said that this is the conclusive test to demonstrate that the fill provides an adequate filter to the core.

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FILTERING CAPABILITY OF GLACIAL TILL SHOULDER FILL USING THE DELGADO PERMEABILITY CRITERIA

Delgado et al (2006) re-examined 340 laboratory tests by others and carried out 348 tests further tests. From the successful tests, those in which the filter trapped the base soil particles and prevented erosion from continuing, he derived the relationship shown on Figure 8, linking the percentage of the base soil passing the 0.075 mm sieve to the filter permeability required to filter it.

It should be noted that the objective was to relate filtering capability to permeability, a hydraulic criterion, as this appears to relate more truly to filter behaviour than geometric grading criteria alone. The 348 Delgado tests show the filtering capacity against a permeability assessed from density through specific regression formulae. Permeabilities were not measured in most of the 340 other tests, and the permeabilities were estimated by applying the Sherard relationship between permeability and D_{15} filter size, as follows:

$$k_{\text{filter}} = 0.35*(D_{15})^2, k \text{ in cm/s, } D_{15} \text{ in mm}$$

This relationship gives permeability values markedly (nearly three times) higher than the Delgado regression formulae.

Using the Delgado line on Figure 8, the required permeability of the filter to trap the core with 80% (of the below 4.75 mm grading) passing 0.075 mm is 2.0E-02 cm/s (2.0E-04 m/s). The permeability of the fill calculated by the Sherard formula with D_{15} of 0.4 mm (see Figure 7) is 5.6E-02 cm/s, more than twice as high as required and the 'simple' approach suggests that the fill would be too permeable to filter the core.

Using the Delgado regression curve for a compacted filter (Figure 9) shows that the permeability is 2E-02 cm/s (0.02 cm/s), exactly the required value, indicating that the fill would filter the core. The permeability of the fill measured in the permeameter was between 1.2E-04 and 6.0E-03 cm/s, lower than the required value and again indicating that the fill would filter the core.

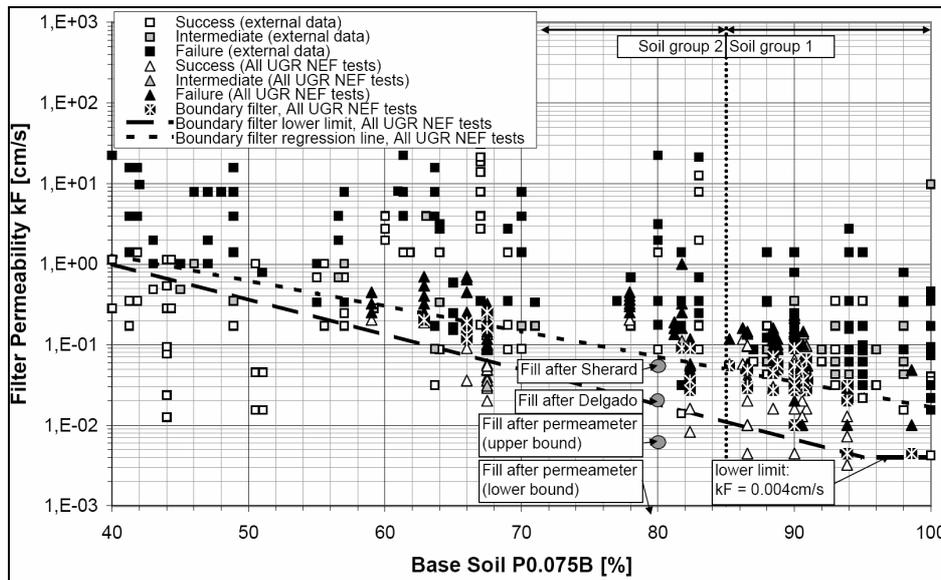


Figure 8: Delgado relationship between % base soil passing 0.075 mm sieve and filter permeability [3]

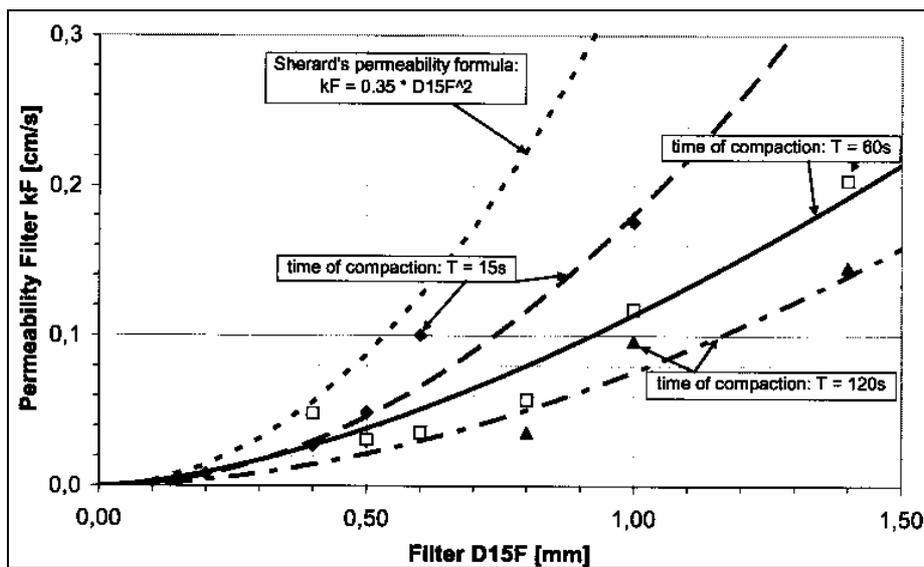


Figure 9: Delgado relationship between D_{15} and permeability of filters [3]

ESTIMATED PROBABILITY OF EROSION USING FOSTER & FELL'S METHOD

Foster & Fell (1999, 2001) use the method of logistic regression, already applied by Honjo & Veneziano (1989), to replace deterministic no-erosion criteria by a probability-based approach. This allows for state-of-erosion criteria being viewed as gradual rather than strict. According to this methodology, the conditional probability of a data point which is

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characterized by a vector of parameters x_i for showing some defined property C, is defined as follows:

$$P(C | x_i) = \frac{e^Z}{1 + e^Z},$$

where e is Euler's number and Z is the general function $Z = \omega_0 + \sum_{i=1}^n \omega_i x_i$. ω_i are weighting factors which are calibrated using available data. In the present case, C is the "showing of erosion" in filter tests, e.g. the NEF test which is used by Foster & Fell (1999) in order to extend existing data bases.

The core material in the present study shows a $pp_{0.075B}=80\%$, i.e. the percentage of grains of the base soil finer than a particle size of 0.075 mm. According to Foster & Fell (2001), this soil is classified as being in Soil Group 2A, ranging from $pp_{0.075B}=35\%$ to $pp_{0.075B}=85\%$, for which no apparent relationship between the base-filter interaction and grain-size characteristics of the base soil can be identified (see also Sherard & Dunnigan, 1989). Therefore, Z in the logistic regression function is only dependent on D_{15F} of the filter. As Foster & Fell (1999) do not give the mathematical definition of Z for soil group 2A, it has to be approximated from the graphical representation in their 1999 paper and the given information is that $P(\text{failure}|D_{15F}=0.82 \text{ mm}) = 0.5$. This approximation gives the function $Z \approx 3 \cdot (D_{15F} - 0.82)$. Introducing the $D_{15F} = 0.2$ mm of the dam investigated in this paper, the probability of erosion is calculated as $P(\text{erosion}|D_{15F}=0.2 \text{ mm}) = 0.13$; and in case of a $D_{15F} = 0.4$ mm the probability increases to 0.22, in both cases not insignificant but low.

SEEPAGE VELOCITIES TO INITIATE EROSION

The initiator of internal erosion is the energy available from the seeping water to erode soil particles from the walls of seepage channels. Both the upstream and downstream shoulders of the dam are in glacial till fill of low permeability which may limit quantity and velocity of water reaching any flow channels through the core. However, the undispersed 'natural' grading of the core show that it has a high silt content and may therefore behave as non-cohesive silt, very vulnerable to erosion because erosion may be initiated at velocities as low as 0.012 m/s (Bridle, 2007). Very approximate flownet analysis examining flow towards a 10 mm high crack at about one-third height of the core into which water flows from the isotropic upstream shoulder and escapes 'freely' along a shorter flowpath through the isotropic downstream shoulder into a downstream drain showed that velocities could be high enough to initiate erosion in all but the lowest permeability fill (1.2E-06 m/s). However, silt is non-cohesive and could not sustain an open crack if saturated, consequently seepage velocities would be much reduced,

BRIDLE

probably too low to cause erosion. Clays will resist high velocities, 0.4 to 1.9 m/s, depending on type (Bridle, 2007).

However, calculation of the hydraulic conditions, by Fry (based on Fry & Blais, 2006, and Fry, 2007) leads to the conclusion that 'tolerable' erosion would occur if the permeability of the fill is $6.0E-05$ m/s (the highest permeability measured in the laboratory permeameter) or less. The erosion would be tolerable because the hydraulic head in the downstream fill would increase and in consequence the difference of hydraulic head through the core (or the hydraulic gradient) would decrease as the discharge flow increased, in consequence the flow velocity would decrease down to a critical value at which erosion would stop, even though total seepage flows of several litres/second could be occurring. The approach shows that the downstream fill filters the core even if erosion has been initiated.

QUANTITATIVE RISK ASSESSMENT

A Quantitative Risk Assessment (QRA) has been carried out using the Interim Guide to Quantitative Risk Assessment for UK Reservoirs (Brown & Gosden, 2004). Unlike other methods (for example Lacasse, Nadim, Hoeg & Gregersen, 2005) which assign arbitrary probabilities to qualitative descriptions such as 'unlikely', 'likely', etc, to an event train to derive probabilities, the QRA Guide produces approximate, but absolute, assessments of the probability of failure of typical British embankment dams based on a comparison of a subject dam's characteristics and performance to the characteristics and performance of similar dams (Brown & Tedd, 2003). As there are over 1,000 'typical British embankment dams', most more than 100 years old, at least 100,000 dam-years of performance data are available, a few of which were poor performances, recorded as incidents, from which probabilities of failure were projected. The method was developed primarily to examine the vulnerability of embankment dams to internal erosion, as other methods were not available.

The total annual probability of failure (AP) of the subject dam was found to be about $2E-06$ (loosely 1 in 500,000-years). It is most vulnerable to internal erosion through the embankment ($1.5E-06$), condition score 2, resulting from a pessimistic scores of 1 each on settlement (of which there has been none in recent years), and a single episode of rapid refill. Erosion alongside the overflow channel and tunnel is less likely ($5E-07$) and failure through overtopping is extremely unlikely ($1E-07$). However, because many lives would be at risk should the dam fail without warning, even this very low overall probability of failure is, to standards recommended by the Health & Safety Executive, only 'tolerable' (in the zone between 'unacceptable' and 'broadly acceptable') as a risk imposed on society. The QRA methodology enables engineers to examine the effects of safety measures to improve

ENSURING RESERVOIR SAFETY

safety and whether the costs of doing so are proportionate. In this case small improvements in monitoring and in emptying capacity would likely be sufficient at proportionate cost to further reduce the probability of failure and improve the safety status to be ‘broadly acceptable’.

CONCLUSIONS AND RECOMMENDATIONS

Although its performance gives no reason for concern, the objective of the investigations is to assess the dam’s ability to resist internal erosion. As it is old, it can be assumed that no large openings exist through which flow could occur at a velocity high enough to initiate erosion and lead to piping and failure. The mechanism envisaged is erosion in a (possibly new) discontinuity through the core, or general erosion if the possibility that the core is silt-sized, as undispersed (natural) samples show, and the question is whether the adjoining glacial till fill would act as a filter, restricting velocity and trapping eroded particles to prevent erosion. The outcomes are summarized in Table 1.

Table 1: Summary of results

Method	Results	How re-assuring?	Remarks
Vaughan & Soares	Filters down to 0.3 - 4.2 microns, but 7-29% dispersed core may be eroded	Not	‘Perfect’ filter, no movement permitted, thought to be conservative. Natural soils are undispersed, silts more easily eroded than clays
Sherard	D ₁₅ 0.4 mm, less than critical 0.7 mm, but grading generally too coarse to filter Category 2 core. Fill passed No Erosion Filter test	Moderately	Sherard filters develop by self-filtering, some particles are eroded and build up a filter in-situ. Fill not dispersive.
Delgado	Permeability of fill, assessed from grading and laboratory results, low enough to filter core	Re-assuring	Uses permeability criterion, but less conservative than Vaughan & Soares, perhaps because self-filtering is assumed, as Sherard
Foster & Fell	Probability of occurrence of erosion not insignificant, but low	Moderately	However, scale of ‘not insignificant’, ‘low’, not clear

BRIDLE

Method	Results	How re-assuring?	Remarks
Bridle	Seepage velocities high enough through 10 mm opening to move silt core in all but lowest permeability fill.	Moderately	Silt core non-cohesive, would not sustain 10 mm opening. Clay would sustain opening, but would not be eroded at velocities generated through it.
Fry	Erosion may initiate, but would be filtered in fill, even with large seepage quantities	Very	Seems to demonstrate what would be expected intuitively in a dam with low permeability, fine fill
QRA	'Tolerable' to 'broadly acceptable' probability of failure	Very	Typical British embankment dams have not shown vulnerability to long term internal erosion, reasons perhaps indicated by results above.

The general conclusion from the results is that methods which analyse the mechanics of erosion in the dam seem to best reflect the actual performance of the dam, which has been demonstrated by the QRA to be likely to be good when compared to the behavior of many similar dams.

The Fry approach and the QRA are very re-assuring about the vulnerability of the dam to internal erosion. The other results are less so but this can be explained by the fact that they are derived from design methods for filters and therefore reflect the very stringent conditions imposed in the experiments used to establish adequately conservative filter design criteria. Filters are usually thin and must operate under high hydraulic gradients and high seepage velocities. Filter design criteria are therefore very conservative. By contrast, because shoulder fill is wide and is commonly, as in this case, similar both upstream and downstream of the core, hydraulic gradients and seepage velocities are reduced thereby reducing the erosive energy of seeping water.

The fill in the subject dam is widely graded and, because it was placed by side tipping, may be segregated with locally coarse and fine variations throughout the shoulders, but because of its geological origins, it is probably unusually uniform when compared to shoulder fill in many typical British dams. Its vulnerability to internal erosion can be examined by simple analyses based on information from laboratory tests on samples collected from trial pits. Borehole investigations are intended to check that the

ENSURING RESERVOIR SAFETY

variability of the fill in the dam shoulders is not so great that the findings to date are invalidated.

However, internal erosion can take several forms and be initiated at ‘singularities’ in fill, of which more are likely to be present in the variable shoulder fill probably present in many ‘typical’ British dams. Application of the principles set out by Fell & Fry (2007), Fry & Blais (2006) and Fry (2007) are beginning to make it possible to rationally examine and analyse the mechanisms and assess the likelihood of the various types of erosion – backward, crack, suffusion or contact – progressing at vulnerable locations because of the loadings applied through the five stages – initiation, continuation, progression, detection, intervention - that may lead to a breach through internal erosion by enlargement of erosion pipes, by loss of freeboard through crest settlement, by slope instability or by unraveling of the downstream slope. The methodology can be applied to erosion through the body of embankment dams and at interfaces with structures – culverts, spillways- through or on dams.

RECOMMENDATIONS

Internal erosion is thought to be a serious threat to typical British dams, similar to overtopping and instability, for which there is guidance. The QRA Guide is the guidance on internal erosion and should be applied at every embankment dam. If it suggests that the dam is vulnerable to erosion, further investigations, progressively making use of more detailed information collected from existing records, local geology, trial pits, boreholes (and equipment capable of drilling on slopes without scaffolding is now available) to carry out analyses of the vulnerabilities, using methods such as those reported by Fell & Fry (2007), to guide decision-making on whether it is necessary to respond to those vulnerabilities by appropriately targeted monitoring or by safety works.

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ENSURING RESERVOIR SAFETY

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