SYNOPSIS. Concrete dam displacements measured by pendulum need to be interpreted to evaluate the dam behaviour. This article presents different statistical models used to evaluate the thermal displacements in concrete dams. The difficulties involved in assessing these displacements are highlighted and possibilities for improvements are presented. A new model based on both water and air temperatures is then detailed and the results obtained for a French dam are presented.

INTRODUCTION

Dam safety is an important issue for dam management. Although the probability of dam failure is very low, such an event would lead to very significant losses. The associated risk is thus very high. Moreover, as the structural vulnerability increases with dam ageing, it is essential to monitor dams to ensure their safety. The structural health diagnosis of large concrete dams is based on monitoring which aims at detecting and quantifying, as soon as possible, the slightest change in dam behaviour.

Monitoring consists of collecting data from instruments and interpreting these measures. The main part of dam surveillance is to analyse gathered data to ensure that the dam is functioning as intended, to detect any possible anomalies, and to warn of any change which could endanger its safety. Data analysis is also a means to better understand the long term behaviour of dams. Since structural responses of dams are influenced by several factors, engineers use different analytical tools to evaluate dam behaviour from collected data.

The analysis of displacement measurements from direct or inverted pendulums represents an important part of concrete dam surveillance. These displacements are influenced by various factors such as hydrostatic
load, thermal effect and time-dependent irreversible phenomena (creep, alkali-aggregate reaction, adaptation, consolidation, damage...). The simplest analysis consists of plotting measured displacements as a function of time. Nevertheless, this type of graph is difficult to analyse because of scatter due to external reversible influences (thermal and filling conditions). Consequently, to provide surveillance of its dams, EDF uses statistical models to separate the influences of the different explicative factors. It is then possible to observe anomalies or irreversible trends. Moreover, the understanding of reversible influences gives important information on the behaviour of the structure.

MODELS IN USE AND POSSIBILITY OF IMPROVEMENT
Displacement measurements are widely influenced by the sequence of thermal conditions and filling conditions. Consequently, direct surveillance of displacements can only be effective by comparison of similar situations (thermal state and retention level). In order to prevent this problem and to generalise comparisons of all situations encountered, EDF uses statistical analytical methods to correct measurements from both thermal and hydrostatic influences (postulated as reversible influences) and thus to highlight irreversible behaviour (figure 1).

Figure 1. Measured and corrected displacements (top left), modelled thermal displacements (top right) and modelled hydrostatic displacements (bottom)

These statistical models are called “surveillance at reconstituted constant conditions”. Two of these models are mainly used to analyse displacements
of concrete dams: HST (Hydrostatic, Seasonal, Time) and HSTT (Thermal HST).

**HST (Hydrostatic, Seasonal, Time)**
The so-called HST statistical analysis method has been developed at EDF by Willm and Beaujoint [1]. In this model the measured displacements are assumed to be the sum of three influences:

- The thermal influence which is modelled as a sinusoidal function of the season only. The seasonal evolution depends on an angle $S$ which is equal to $0^\circ$ on 1 January and to $360^\circ$ on 31 December.
- The hydrostatic influence, modelled as a fourth degree polynomial function of the retention level $Z$.
- The irreversible influences, if any, modelled by a combination of a polynomial function (creep, swelling and quick evolutions) and an exponential function of the time $t$ (consolidation, adaptation ...)

Recorded dam displacements $Y_0$ are thus modelled by the following expression:

$$
Y_0 = a_1 + a_2 \cdot t + a_3 \cdot t^2 + a_4 \cdot t^3 + a_5 \cdot t^4 + a_6 \cdot e^{-t/\tau} \\
+ a_7 \cdot Z + a_8 \cdot Z^2 + a_9 \cdot Z^3 + a_{10} \cdot Z^4 + a_{11} \cdot \cos(S) \\
+ a_{12} \cdot \sin(S) + a_{13} \cdot \cos(2S) + a_{14} \cdot \sin(2S) + \varepsilon
$$

(1)

The coefficients $a_1$ to $a_{14}$ and the parameter $\tau$ are adjusted on measurements by the least square method. $\varepsilon$ represents the scattering of the model which contains the uncertainties of both the experimental measurements and the model.

The results obtained with HST over several decades of displacement analysis of concrete dams have confirmed the relevance and robustness of this method. The main advantage of this model is its simplicity and the fact that it does not need temperature measurements to account for the thermal influence.

However, the main limitation of this model is that the thermally induced displacements are modelled as an annual periodic response. Approximating the thermal effect by an average seasonal distribution gives relevant results in the vast majority of cases. Nevertheless, since the real temperature evolution is not accounted for in Eq. (1), the performance of HST is not always sufficient, in particular for time periods colder or warmer than seasonal average. Moreover, HST is not able to capture a drift in the thermal state of the structure (global warming). Consequently, if such a drift occurs it could be interpreted as an irreversible effect.
HSTT (Thermal HST)
Since 2004, an improvement of the HST model, called HSTT (Thermal HST) [2] is in use for arch dams. This model completes the seasonal function by a corrective term which takes into account the delayed effect of the structure to daily temperature difference between the one recorded in the air and the average seasonal one for the time of the year. As for hydrostatic and time-dependent effects, they stay unchanged compared to the HST model. In this model, the major factor in thermally induced displacement is assumed to be the air temperature. For each day $j$, the air temperature $\Theta(j)$ comprises the sum of the average seasonal air temperature $N(j)$ (modelled by a sinusoidal function with a period of one year) and the difference $\Delta\Theta(j)$ between the daily recorded temperature and this seasonal temperature.

\[
\Theta(j) = N(j) + \Delta\Theta(j)
\]

The principle of HSTT is described in table 1.

Table 1. Principle of HSTT

<table>
<thead>
<tr>
<th>Air temperature</th>
<th>Average seasonal temperature at day $j$ $N(j)$</th>
<th>Deviation at day $j$ $\Delta\Theta(j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal inertia of the structure</td>
<td>Thermal corrective function delayed specific HSTT</td>
<td></td>
</tr>
<tr>
<td>Thermal displacement</td>
<td>Seasonal function of period 1 year with shifted phase</td>
<td></td>
</tr>
</tbody>
</table>

The delayed elevation $\Delta\Theta_R$ of the mean temperature of the structure induced by the air temperature deviation $\Delta\Theta$ is calculated by convolving the deviation $\Delta\Theta$ with the impulse response of the structure. This convolution product can be expressed as a recurrence formula:

\[
\Delta\Theta_R(t + dt) = \Delta\Theta(t + dt)(1 - e^{dt/T_0}) + \Delta\Theta_R(t)e^{-dt/T_0}
\]

In Eq. (3) the parameter $T_0$ is the characteristic time of the structure representing its thermal inertia which depends on the geometrical and thermal properties of the structure.

The thermally induced displacements due to the deviation $\Delta\Theta$ are then assumed to be proportional to the delayed elevation $\Delta\Theta_R$ of the mean temperature with a factor of thermal sensitivity $K$.

In the HSTT model, the recorded displacements $Y_0$ are modelled by:

\[
Y_0 = a_1 + a_2 \cdot t + a_3 \cdot t^2 + a_4 \cdot t^3 + a_5 \cdot t^4 + a_6 \cdot e^{-t/\tau} + a_7 \cdot Z + a_8 \cdot Z^2 + a_9 \cdot Z^3 + a_{10} \cdot Z^4 + a_{11} \cdot \cos(S) + a_{12} \cdot \sin(S) + a_{13} \cdot \cos(2S) + a_{14} \cdot \sin(2S) + K\Delta\Theta_R + \epsilon
\]

In Eq. (4) the coefficients $a_1$ to $a_{14}$, $K$ and $T_0$ are adjusted on measurements by the least square method. $\epsilon$ represents the scattering of the model which
contains the uncertainties of both the experimental measurements and the model.

This improvement results in a better correction of the thermal influence for colder or warmer time periods than usual. The HSTT model can explain in the vast majority of cases the displacements at the crest of thin arch dams. For example, the heat wave of 2003 in France generated important displacements which were not explained by the HST model and are better explained by the HSTT model. Besides, compared to HST, HSTT reduces the scatter of corrected data and has a better explanatory quality. This reduced scatter allows an earlier detection of anomalies and thus the structure behaviour can be better diagnosed.

**Possibilities of improvement**

It is well known that temperature variations are one of the most important influences that affect the recorded displacements. Thus, this thermal effect has to be modelled as accurately as possible. There are different thermal influences which act on dam displacements: air temperature, water temperature, heat transfers from foundations, solar radiation...

The HST model takes into account all these different phenomena in only one seasonal function. In the case of HSTT, the deviation of air temperature compared to the average seasonal value is separated from the other influences. Nevertheless, in both models, the not all the different thermal influences are explicitly taken into account. An important possibility for improvement for the model is to separate all these influences which are probably not well modelled by a unique seasonal function.

Moreover, in the HST and HSTT models the seasonal and the hydrostatic influences are considered to act separately, whereas they are in reality coupled. Indeed, the influence of water temperature is dependent on the retention level. Besides, the retention level often follows a cyclic evolution due to supply and demand requirements. If this cycle of retention level variation is in phase with the seasonal temperature variation, hydrostatic and thermal effects are correlated and it is then difficult to separate them.

**USE OF TEMPERATURE MEASUREMENTS**

As detailed above, the HST model does not need any temperature measurements to evaluate thermal effect. The use of temperature measurements in the models allows the thermal state of the structure to be better taken into account and permits more precise evaluation of thermally induced displacements. Several methods are available to use temperature measurements in statistical models.

In the HSTT model, an improvement has been incorporated by integrating air temperature measurements. The elevation of the mean temperature
within the structure is determined by solving a direct one-dimensional heat transfer problem with recorded air temperature as a boundary condition.

It is also possible to utilise internal concrete temperatures if representative internal thermometric data are available. One of the advantages of this method is to account for all external influences (radiation, air temperature, water temperature) because internal thermometers directly measure the effect of these influences. However, one issue is the loss of information when using embedded thermometers. Indeed, the high frequencies of the thermal signal do not penetrate deeply into structures and hence are not captured by thermometers if they are too far from the surface. For example, a signal with a time period of one day will have a depth of penetration of approximately 0.50m. If the thermometer is located at more than 0.50m from the surface it will not measure the daily variations of these external thermal influences. Nevertheless, as these high frequency signals impact a relatively small part of the structure, their effect on global displacements should be low.

The first approach proposed is to directly use the measured temperature in the statistical formulation. This is the case in the HTdT model (hydrostatic, direct temperature, time) proposed by Weber [3]. In this model temperature measurements inside the dam are new explicative variables that replace the seasonal function. A general formula of this type of model is given by Eq. (5) where $n_T$ is the number of thermometers used and $T_i$ is the temperature measurements from thermometer $i$. It is preferable to use the measurements for several elevations near both the upstream and the downstream surfaces and in the middle of the cross section in order to identify the different thermal influences as well as possible.

$$Y_0 = a_1 + a_2 \cdot t + a_3 \cdot t^2 + a_4 \cdot t^3 + a_5 \cdot t^4 + a_6 \cdot e^{-t/\tau} + a_7 \cdot Z + a_8 \cdot Z^2 + a_9 \cdot Z^3 + a_{10} \cdot Z^4 + \sum_{i=1}^{n_T} b_i \cdot T_i + \epsilon$$

A better way to utilise concrete temperatures is to use them to reconstitute the thermal field inside the structure and to calculate displacements from this thermal field. In this case a one-dimensional inverse problem is solved to obtain the temperature at the surface from two thermometers inside the structure and then the thermal field is rebuilt from the surface temperatures. This method has been treated by Léger and Leclerc [4] in the case of a periodic thermal signal and by extension for any transient signal by adding trailing temperatures at the end of the signal. The reconstituted thermal field is deconstructed along different sections by average and linear temperature differences which are used in the so-called HTT (Hydrostatic, Temperature, Time) model. The formulation of this model is given by Eq. (6) where $n_{Tsec}$ is the number of sections where the one dimensional thermal field $T(x)$ is computed from thermometers located in the section, $T_{m,i}$ and $T_{g,i}$ are respectively the mean and linear difference temperatures of
the section \(i\) and \(T_{\text{ref}}\) is the reference temperature (long term average concrete temperature). In this model the thermally induced displacements are separated into two parts. One is proportional to the elevation of the mean temperature from the reference temperature, and the other is proportional to the linear difference temperature. As in the HT\(_d\)T model, the sections have to be properly chosen to account for all the thermal influences. However, in practice, the chosen sections are the ones which contain the thermometers.

\[
Y_0 = a_1 + a_2 \cdot t + a_3 \cdot t^2 + a_4 \cdot t^3 + a_5 \cdot t^4 + a_6 \cdot e^{-t/\tau} \\
+ a_7 \cdot Z + a_8 \cdot Z^2 + a_9 \cdot Z^3 + a_{10} \cdot Z^4 \\
+ \sum_{i=1}^{n_{\text{sec}}} (b_i \cdot (T_{m,i} - T_{\text{ref}}) + c_i \cdot T_{g,i}) + \varepsilon
\]

(6)

A more rigorous approach to calculate thermal displacements from embedded thermometers has been proposed by Weber, Perner and Obernhuber [5]. In this method, the temperatures at the boundaries are calculated from internal thermometers with a one-dimensional inverse heat transfer problem. These temperatures are then extrapolated to the entire upstream and downstream surface and convolved with impulse responses to compute the mean and the gradient of the temperature field across the structure. The thermal displacements are not calculated statistically as before but employ the thermo-elastic reciprocal theorem. To calculate the displacement at a given point in a given direction, one needs to know the stress first invariant field \(\Theta = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}\) in the structure due to a unit force applied at this point and in this direction. This stress field can be obtained by a finite element simulation. The thermal displacements \(\delta\) due to the thermal field \(T\) can be calculated by:

\[
\delta(t) = \alpha \cdot \int_Y \Theta \cdot T(t) \cdot dx
\]

(7)

In this expression \(\alpha\) is the thermal expansion coefficient. To simplify Eq. (7) it is assumed that the stresses vary linearly over the thickness for the applied unit force. This approximation is reasonable for a thick enough structure and far enough from the foundations. With this approximation the integral can be written in the radial direction as Eq. (8) where \(\Theta_M, \Theta_D, T_M\) and \(T_D\) are the mean and the gradient of the first invariant and the mean and the gradient of the temperature over the thickness respectively.

\[
\int_0^L \Theta \cdot T(t) \cdot dV = L \cdot \Theta_M \cdot T_M(t) + \frac{L}{3} \cdot \Theta_D \cdot T_D(t)
\]

(8)

Eq. (8) justifies that only the mean and the gradient of the temperature field are necessary to compute thermal displacements if the structure is thick enough. The mean and the gradient of the temperature are then considered to be constant over horizontal arches to compute the integral along an arc of length \(s\) (Eq. 9).
The variables $M(h)$ and $D(h)$ give respectively the influences of the mean and the gradient of the temperature at the dam height $h$ on the thermal displacements where the unit force has been applied. The final thermal displacements can be obtained by integrating Eq. (9) over the height.

A difficulty in the use of embedded thermometers is the necessity to use several measurements to correctly reconstitute the thermal field. Indeed, water temperature depends on the depth; solar radiation is not homogeneous on the impacted surface (shade effect, and orientation of the surface); and near the foundations the measurements are highly influenced by the conductive heat transfer from the rock.

NEW MODEL AND APPLICATION

Based on the different ideas set out above, a new model has been developed at EDF. This model accounts for air and water temperature by solving a one-dimensional heat transfer problem at several elevations of the dam with water and air temperature signals at the boundaries. A seasonal function is also used to take into account the radiative effect.

Impulse responses

For a one dimensional semi-infinite medium starting from $x=0$, the structural response to an impulse (Dirac) of weight $T_{up}$ is given by Eq. (10), where $t$ is the time, and $a$ is the diffusivity of the medium.

\[
T_{\text{semi-infinite}}(x, t) = \frac{T_{up} x}{2 \sqrt{\pi} a t} \cdot e^{-\frac{x^2}{a^2 t}}
\]

To obtain the solution for a finite medium of length $L$, one needs to superpose an infinite number of semi-infinite solutions in order to satisfy the boundary conditions at $x=0$ and $x=L$. The mean temperature for the finite medium can then be expressed as:

\[
T_{\text{mean}}(x, t) = \frac{1}{L} \cdot \sum_{n=0}^{\infty} (-1)^n \int_{nL}^{(n+1)L} T_{\text{semi-infinite}}(x, t) \, dx
= \frac{T_{up} \sqrt{\pi}}{L \cdot \sqrt{\pi t}} \cdot \left(1 + 2 \cdot \sum_{n=0}^{\infty} (-1)^n \cdot e^{-\frac{n^2 \pi^2}{4 \cdot a^2 t}}\right)
\]

The expression of Eq. (11) is the mean temperature response in a one dimensional wall of length $L$ to an impulse of weight $T_{up}$ on one side of the wall. By extension, Eq. (12) provides the mean temperature response in the
wall for an impulse of weight $T_{up}$ on one side and $T_{do}$ on the other side of the wall:

$$T_{mean}(x, t) = \frac{(T_{up} + T_{do}) \cdot \sqrt{\alpha}}{L \cdot \sqrt{\pi t}} \cdot (1 + 2 \cdot \sum_{n=1}^{\infty} (-1)^n \cdot e^{-\frac{n^2 L^2}{4 \cdot a \cdot t}})$$

Consequently, to obtain the mean temperature in the one-dimensional medium of length $L$ for any signals on the two sides of the medium, one needs to convolve the mean of the two signals $(T_{up} + T_{do})/2$ by the impulse response given by:

$$T_{mean}(x, t) = \frac{2 \cdot \sqrt{\alpha}}{L \cdot \sqrt{\pi t}} \cdot \left(1 + 2 \cdot \sum_{n=1}^{\infty} (-1)^n \cdot e^{-\frac{n^2 L^2}{4 \cdot a \cdot t}}\right)$$

It is worth noting that the impulse response for the mean temperature (Eq. (13)) is the same as the one used and demonstrated by Weber, Perner and Obernhuber [5].

Concerning the gradient temperature in the one-dimensional medium of length $L$, it can be calculated for any signal on the two sides of the medium by convolving the signal $(T_{up} - T_{do})/2$ by the impulse response [5] given by:

$$T_{grad}(x, t) = \frac{6 \cdot \sqrt{\alpha}}{L \cdot \sqrt{\pi t}} \cdot \left(1 + 2 \cdot \sum_{n=1}^{\infty} e^{-\frac{n^2 L^2}{4 \cdot a \cdot t}}\right) - \frac{12 a}{L^2}$$

**Air and water temperatures**

In this model the dam is sliced up into 10 layers (Fig. 2). Each layer is considered as a one-dimensional medium and the mean and the gradient of the temperature field are individually computed from air and water temperatures using convolving products with the previously seen impulse responses (Eq. (13) and (14)).

![Figure 2. Dam slicing into 10 parts along the height.](image)

For each layer, the signal $T_{do}$ is the daily air temperature signal $T_{air}$ and, depending whether the layer is above or below the water level, the signal $T_{up}$ is the daily air temperature signal $T_{air}$ or the daily water temperature signal $T_{water(d)}$ at depth $d$, respectively.

The daily air temperature signal $T_{air}$ is determined by air temperature measurements. As water temperatures are not monitored along the reservoir
depth, a model is used to predict the signal $T_{water}(d)$. The water temperature model is an empirical model based on two measurement surveys, one in summer and one in winter. The temperature at the water surface is modelled by Eq. (15) where the angle $S_w$ varies linearly between 0° (25 January) and to 360° (24 January the following year).

$$T_{surf}(t) = 13 - 8 \times \cos(S_w)°C$$

Temperatures at different depths under the water surface are calculated by means of Eq. (16) where $d$ is the depth under the water surface (in metres).

$$T_{water}(d, t) = 5 + (T_{surf}(t) - 5) \cdot e^{-d/13°}C$$

Statistical model and results

As shown in Eq. (9), with some assumptions, at a given elevation the contribution to thermal displacements is proportional to the mean and the gradient of the temperature field calculated for this elevation. The coefficients of influence $M(h)$ and $D(h)$ are adjusted statistically for each layer. To account for solar radiation a seasonal function has been added to the model. The formulation of the model is given by Eq. (17) where $nL$ is the number of layers, $T_{mean,i}$ and $T_{grad,i}$ are respectively the mean and the gradient of the temperature field calculated for the layer $i$.

$$Y_0 = a_1 + a_2 \cdot t + a_3 \cdot t^2 + a_4 \cdot t^3 + a_5 \cdot t^4 + a_6 \cdot Z + a_7 \cdot Z^2 + a_8 \cdot Z^3 + a_9 \cdot Z^4 + a_{10} \cdot \cos(S) + a_{11} \cdot \sin(S) + a_{12} \cdot \cos(2S) + a_{13} \cdot \sin(2S) + \sum_{i=1}^{nL} (b_i \cdot T_{mean,i} + c_i \cdot T_{grad,i}) + \epsilon$$

Figure 3. Comparison between displacement measurements with pendulum and displacements calculated by the model.

It is worth noting that the modelled displacements reproduce the observed displacements well (Figure 3), particularly when these displacements are greater than usual (2003 heat wave for example). These unusual
displacements are due to unusual thermal conditions and can be observed in Figure 4 which compares thermal displacements modelled by HST, HSTT and the new model. When the thermal displacements of HST cannot take into account these unusual thermal conditions, those of HSTT and of the current model are more representative of the real thermal conditions.

Figure 4. Comparison between thermal displacements calculated by HST, HSTT and the new model

CONCLUSIONS AND PERSPECTIVES
Different methods are used to estimate thermally induced displacements in concrete dams. The use of temperature measurements appears to add important information for statistical models but it is not easy to utilise them correctly. The new model presented in this paper, although it is more complex than HSTT (it adds water temperature measurements, separates the influences of several layers and take into account the gradient effect), does not improve significantly HSTT. The scatter is slightly reduced and the correlation coefficient is equal to 0.9825 instead of 0.9803 for HSTT. Nevertheless, different improvements could be considered:

- The air temperature signal is at a high frequency and could be incorporated in a smoother way.
- The concrete temperature signal could be used in order to obtain a better assessment of the thermal field inside the structure.
- Radiative effects and water temperature effects could be added in an improved way, using measurements.
- The hypothesis of uni-axial conduction is not confirmed, in particular near the foundations.
To improve the statistical model it is also possible to use finite element simulations to evaluate and quantify all thermal effects (solar radiation, convection, transfer from foundations, etc.) and to validate the hypotheses and better assess the thermo-mechanical fields of the statistical approaches. Finally, it is possible to recapture statistical models, to clarify and to improve them in order to estimate more accurately the behaviour of arch dams.

REFERENCES


